

Comment on “Proton Polarization Shifts in Electronic and Muonic Hydrogen”

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Abstract

We demonstrate that criticisms concerning our work nucl-th/9704043, made in the recent preprint hep-ph/9903352, are unfounded. The improvements over our results claimed in hep-ph/9903352 are in fact spurious, being based mainly on misunderstandings and on dubious theoretical models.

In [1] (see also [2]) we found the correction to the hydrogen Lamb shift due to the proton electric and magnetic polarizabilities. This our result was criticized recently in [3]. One cannot agree with these criticisms.

To make our objections more comprehensible, let us present at first the key points of our derivation [1]. We started it with the electron-proton forward scattering amplitude:

$$T = 4\pi i\alpha \int \frac{d^4k}{(2\pi)^4} D_{im}(k) D_{jn}(k) \frac{\gamma_i(\hat{l} - \hat{k} + m_e)\gamma_j}{k^2 - 2lk} M_{mn}. \quad (1)$$

Here D_{im} , D_{jn} are the photon propagators, we use the gauge $A_0 = 0$ for virtual photons; $l_\mu = (m_e, 0, 0, 0)$ is the electron momentum. The nuclear-spin independent Compton forward scattering amplitude, which is of interest to us, can be written as

$$M = \bar{\alpha}(\omega^2, \mathbf{k}^2) \mathbf{E}^* \mathbf{E} + \bar{\beta}(\omega^2, \mathbf{k}^2) \mathbf{B}^* \mathbf{B} = M_{mn} e_m e_n^*, \quad (2)$$

where $\bar{\alpha}$ and $\bar{\beta}$ are the nuclear electric and magnetic polarizabilities, respectively. Both expressions are exact (to the discussed order in the fine-structure constant α).

We performed the integration in formula (1) with the logarithmic accuracy, i.e., neglecting in the result terms on the order of unity as compared to $\ln \bar{E}_p/m_e$. Here \bar{E}_p is the typical excitation energy for the proton, m_e is the electron mass. It should be emphasized that $\ln \bar{E}_p/m_e$ is not just a mere theoretical parameter, it is truly large, about 6–7, which makes the logarithmic approximation quite meaningful quantitatively. This is the only theoretical approximation made in [1].

The resulting effective operator of the electron-proton interaction (equal to $-T$) can be written in the coordinate representation as

$$V = -\alpha m_e [5\bar{\alpha}(0) - \bar{\beta}(0)] \ln \frac{\bar{E}}{m_e} \delta(\mathbf{r}). \quad (3)$$

Now, we go over to the discussion of some statements made in [3].

1. The assertion of [3]: “For light electronic and muonic atoms the energy shift due to virtual excitations can be written as an integral over the forward virtual Compton amplitude” is incorrect. In fact, this integral fixes (in the logarithmic approximation) the sum of the polarizabilities $\bar{\alpha}(0) + \bar{\beta}(0)$. To calculate the correction, one needs also the data on the backward Compton scattering, which fix $\bar{\alpha}(0) - \bar{\beta}(0)$.

2. The abstract and section 1. of [3] are written in such a way that an impression may arise that higher multipoles and transverse excitations have been neglected in our paper [1].

In fact, higher multipoles are accounted for in formula (3), as far as they contribute to the total photoabsorption cross section $\sigma_\gamma(\nu)$ and to the backward Compton scattering.

As to transverse excitations, they are also accounted for in formula (3). Indeed, in the combination $5\bar{\alpha}(0) - \bar{\beta}(0)$, the contribution $4\bar{\alpha}(0)$ is due to the Coulomb interaction, but $\bar{\alpha}(0) - \bar{\beta}(0)$ is due to the exchange by transverse quanta, i.e., to the magnetic interaction of convection currents [4].

3. The shift of the mean excitation energy \bar{E}_p to 410 MeV, as advocated in [3], from our estimate 300 MeV, changes the result by 5% only, which is well within our estimate 15% for the accuracy of the used logarithmic approximation. Obviously, such an improvement is not worth discussion.

4. We do not think that the mentioned estimate 15% for our theoretical accuracy can be improved by using “the simple harmonic oscillator quark model” of the nucleon, even if the quark mass M_{quark} in it is taken with four digits (see the caption to Table 1 in [3]).

5. The only, marginal, improvement over our results made in [3], is related to the use therein of more precise experimental data on the total photoabsorption cross section $\sigma_\gamma(\nu)$ [5]. The value for the sum of the polarizabilities obtained from these data in [6] is

$$\bar{\alpha}_p(0) + \bar{\beta}_p(0) = (13.69 \pm 0.14) \times 10^{-4} \text{ fm}^3. \quad (4)$$

(Somewhat different number, without error bars, is presented in [3]: $\bar{\alpha}_p(0) + \bar{\beta}_p(0) = 13.75 \times 10^{-4} \text{ fm}^3$.) We used in [1] an older value for $\sigma_\gamma(\nu)$, which resulted in

$$\bar{\alpha}_p(0) + \bar{\beta}_p(0) = (14.2 \pm 0.5) \times 10^{-4} \text{ fm}^3.$$

However, the uncertainty of our result originating from experimental error bars is strongly dominated by the errors in the experimental data on the backward Compton scattering which give

$$\bar{\alpha}_p(0) - \bar{\beta}_p(0) = (10.0 \pm 1.8) \times 10^{-4} \text{ fm}^3.$$

Thus, using (4) practically does not change the corresponding error, 7 Hz, of our final result. As to the central value of our result for the discussed correction to the hydrogen 1S state, it changes due to (4) slightly, from -71 Hz to -70 Hz, well within the mentioned uncertainty of ± 7 Hz.

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